# Written Exam, Department of Economics, summer 2019 Economic Growth

(3-hour open/closed book exam)

June 4, 2019

Answers in English only. This exam consists of 7 pages in total

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- leave the examination
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam

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- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

# Question 1: Essay questions

#### **1.a**

In the Russian Roulette model in Jones (2016): Life and Growth, explain how the optimal growth path is affected if innovation in rare cases lead to disasters. How does your answer depend on assumptions about relative risk aversion?

Answer: In the model, agents will undertake R&D if the expected utility gain from doing so is positive. To forces are at play. R&D will cause consumption to rise by g percent with probability  $1 - \pi$ , and with probability  $\pi$  cause a disaster that kills the agent. If the agent decides not to conduct R&D, consumption will stay flat. Agents will therefore undertake R&D if the expected utility of doing so, i.e.,  $(1 - \pi) u ((1 + g) c_t)$ , is higher than the expected utility of doing nothing, i.e.,  $u (c_t)$ .

By how much an agent prefers a safe outcome relative to a risky one, depends on risk preferences. In the model, the are three possibilities: If agents are relatively risk taking, their preference for riskyness increases as their income grows, and we will end up in an equilibrium in which R&D is preferred in perpetuity. The consequence is exponential growth.

If, however, agents are relatively risk averse, their appetite for risk will decline as their income grows. In the context of the model, the value of staying alive will dominate if the utility of consumption is large. At some point, consumption therefore reaches a level where agents are so rich that they will prefer not to conduct R&D, and growth will cease.

In the special case where risk preferences are such that risk taking neither rises nor declines with income, we get the same result as the case with risk averse agents. The reason is that the value of staying alive still increases with income.

# 1.b

Explain why Google, despite being one of the world's leading companies, has a relatively small impact on GDP. Which of Google's activities count towards higher GDP, and which activities do not.

Answer: Most of Google's revenue comes from advertising, which in the national

accounts is an intermediate good. GDP only counts goods and services that are bought by the final user, and not intermediate goods. Some of Google's activities do enter the national accounts, notably investments in office space, server parks, and (recently added to the GDP statistics) research and development. The services provided by Google's search engine and other products may also indirectly affect GDP if they increase the productivity of other firms.

# 1.c

How does Chinese import competition affect European firms? Can we, based on the available empirical evidence, conclude that China's accession to the WTO has been good for economic growth in the EU? Has it been good for welfare?

Answer: The empirical results in Bloom et al (2016) shows that surviving firms increase patenting activity, ICT use, and TFP in reaction to competition from china. However, the results also show that some firms, notably the ones with low productivity, go out of business with negative effects for their employees, and potentially for the economy. We cannot from the result infer how China's entry into the WTO has affected the aggregate economy. The answer depends on whether the workers laid off from the firms going out of business are absorbed by the surviving firms. In the long run, it seems reasonable that they will find new jobs, in which case increased import competition is good for growth. Another positive effect is that the lower prices of goods imported from China increase the purchasing power of European consumers.

Whether import competition increases welfare depends on whether the short run negative welfare impact from unemployment is offset by the cheaper Chinese goods, and by the long term reallocation towards surviving firms. Unemployment benefits might mitigate the negative effect on welfare, as the results of Aghion et al (2016) on the reading list shows in the context of the US. And given that unemployment benefits in Europe are relatively generous, it would not be unreasonable to think that import competition has benefitted Europe, both economically and in terms of welfare. But of course, we cannot draw strong conclusion based on the available evidence. Evidence suggests that economic growth across countries are highly interdependent. Suppose we think of the world economy as comprising a leader nation, being the center of innovation, and a number of follower countries in terms of technology. Please, explain how a positive productivity shock in the final goods producing sector of the leading nation may affect growth in the rest of the world.

Answer: In the context of technology diffusion we have studied theories where countries are linked via the cost of immitation. The standard assumption is that the cost of immitation rises as the follower grows closer to the leading nation. A positive shock to productivity in the final goods sector of the leading nation will instigate greater demand for intermediate goods, and thereby greater profits from innovation. As a result, the return on R&D rises which raises growth in the leading country, provided the shock is permanent. This acceleration leads to the emergence of a knowledge gap between leader and follower, which reduces the cost of immitation and thus makes it profitable to increase R&D. As a result, growth in the follower country also increases.

#### **1.e**

Suppose a group of countries establish a "common market", which ensures a fully integrated labor market and more competition overall. Imagine these are the only effects of the common market. Will the common market nessessarily increase growth in the member state countries according to the Aghion-Howitt model? Please, explain why/why not.

Answer: No, not nessesarily. The Aghion-Howitt model contain a scale effect property: More people means more ideas. As a result, a larger labor force means more R&D and thus growth. However, if the extent of product market competition intensifies this may work to lower growth. The reason is that lower profit margins reduces profits and thereby the incentive to conduct R&D.

**1.d** 

Statement: "One should subsidize Research and Development!". Do you agree or disagree? Please explain why.

Answer: In the end the answer is: "it depends.". The central argument in favor of an affirmative answer is that R&D involves intertemporal externalities. Simply put, improvements in the stock of knowledge enhances the productivity of future scientists, thereby lowering the costs of innovation. This effect is not internalized by the market, for which reason equilibrium R&D may be sub-optimally low. However, if "fishing out effects" are present this argument is weakened. In principle the intertemporal externality could turn negative in which case R&D should be taxed! Another argument that may lead to a similar conclusion, i.e. R&D should NOT be subsidized, is found in the context of Schumpeterian growth theory. In this body of literature innovation leads to creative destruction whereby incumbent producers are replaced following new innovations. As this process lowers the return to R&D, a fact that the market does not internalize, R&D may in fact be excessive. A further complication arises if we consider countries that perform R&D in order to catch-up with leading countries. If, as may be considered plausible, the cost of imitation increases as the country moves closer to the leading country in productivity terms, innovation today will tend to exert a negative externality on future imitators. Hence, once again, R&D subsidies are not well founded. In the end, therefore, it is a case-by-case consideration whether indeed subsidies should be implemented.

# Question 2: Endogenous growth through factor accumuation

Consider a representative firm i, which employs the production technology

$$Y_{it} = A_t K^{\alpha}_{it} L^{1-\alpha}_{it},$$

where

$$A_t = \bar{A} K_t^\beta. \tag{2.1}$$

Y is output, A is productivity, K is capital and L is labor. We assume the total labor force is constant over time  $L \equiv \sum_{i} L_{it}$ .

#### **2.**a

Provide an interpretation of equation (2.1).

Answer: The equation can reasonably be interpreted to capture learning-by-doing. That is, past productive experience creates productive knowledge (A). More specifically, it is assumed that past investment intensity creates learning gains – a potentially more reasonable specification would involve past productive experience captured by say output. It is also assumed that individual firms capture the knowledge gains from investment efforts in any other firm in the economy. Hence, we are dealing with "external learning", and it is assumed that knowledge created by learning spills over completely. One may observe that evidence does exist in favor of external learning of the sort implied by the equation. The work of Thompson and Thornton, discussed at the lectures, on shipbuilding in the US during WWII suggests that knowledge indeed did spillover from one yard to the next, and even that productive experience on different ships apper to have supported productivity.

### 2.b

Assume competitive markets and that firms are profit maximizing. Denote the real rate of interest by r and the real wage by w. We disregard capital depreciation, which is therefore is put at zero. (a) Derive the aggregate production function. (b) Under which parameter restriction is endogenous growth feasible?

Answer: (a) Profit maximization provides the standard first order conditions

$$r = \alpha \frac{Y_{it}}{K_{it}}$$

$$w = (1 - \alpha) \frac{Y_{it}}{L_i}$$

We observe that all firms will choose the same capital-labor ratio,  $\frac{K_{it}}{L_{it}} \equiv k_{it} = k_t$  for all *i*. Aggregate output is

$$Y_{t} = \sum_{i} A_{t} K_{it}^{\alpha} L_{it}^{1-\alpha} = A \sum_{i} k_{it}^{\alpha} L_{it} = A k_{t}^{\alpha} \sum_{i} L_{it} = A K^{\alpha} L_{t}^{1-\alpha}.$$

Finally, inserting equation (2.1) we obtain

$$Y = \bar{A}K^{\beta}K^{\alpha}L^{1-\alpha}.$$

(b) Endogenous growth requires constant returns to scale in the reproducible factor of production. Hence  $\beta + \alpha = 1$ , so  $\beta = 1 - \alpha$  is required.

#### 2.c

Assume the production technology is as stated above, and that the condition for endogenous growth to arise has been imposed. Assume time is continuous. Consumers derive utility from consumption over an infinite horizon. Specifically  $\int_{t=0}^{\infty} \ln(c_t) e^{-\rho t} dt$ . As noted above, total population is constant, and there is no capital depreciation. It can be shown that it is optimal for consumers to allow consumption to follow  $\dot{c}/c = r - \rho$ . (a) Derive the growth rate of the economy along a balanced growth path. (b) Provide three (3) testable implications, and discuss their empirical relevance.

**Answer:** (a) The presents model is of the AK variety, since  $Y = \bar{A}L^{1-\alpha}K$ . This means the model exhibits balanced growth (which means all endogenous variables grow at the same rate) and there is no transitional dynamics. Consequently, all we need to do is insert the equilibrium real rate of return into the consumption Euler. Using the first order condition from the profit maximization problem of the firm, along with the aggregate production function provides  $r = \alpha \bar{A}L^{1-\alpha}$ , which means the growth rate is

$$\gamma = \alpha \bar{A} L^{1-\alpha} - \rho.$$

(b) (i) Scale matters: larger economies should grow faster than smaller economies. This prediction has not found much support in the data. (ii) The growth rate is independent of the level of GDP (per capita). That is, there is an absence of conditional ( $\beta$ ) convergence. This too is a potentially problematic prediction. (iii) Countries where individuals are more patient (smaller  $\rho$ ) should be more prosperous in the long run. Recent evidence that provides data on (experimentally determined) time preferences appear to support this prediction. (iv) With some math one can show that the model also implies that growth is an afine transformation of the investment rate. This implies that there should be a strong link between the investment rate and the growth rate. This is a potentially problematic prediction: growth and investment rates are not that tightly linked. (v) TFP and capital accumulation is perfectly correlated. While capital accumulation and TFP are indeed positively correlated across countries, this implication is arguably too strong. Note that only 3 predictions are required.

# $\mathbf{2.d}$

The social planners optimally chosen growth rate, given the assumptions above, is

$$\gamma = \bar{A}L^{1-\alpha} - \rho$$

Please, (a) explain why this solution differs from the decentralized solution derived above, and (b) sketch a policy that is ensures the market solution coinsides with the planner solution.

Answer: (a) The key distinction lies in the presence of  $\alpha$ . The parameter reflects that in the market producers do not internalize the fact that capital accumulation leads to learning gains, which enhances productity. As a result, the private return to investments falls short of the social return, which arises when the planner takes the externality into account. (b) Since capital accumulation is associated with positive externalities one should subsidize investments. The subsidies could be financed by lump sum taxes, or, if labor supply is exogenous, a tax on consumption.

# Question 3: House prices and city size

Consider the model in Hsieh and Moretti (2019): Housing Constraints and Spatial Misallocation. In the model, output in city i is produced according to:

$$Y_i = A_i L_i^{\alpha} K_i^{\eta} T_i^{1-\alpha-\eta}, \tag{3.1}$$

where  $A_i$  is local TFP,  $L_i$  is labor,  $K_i$  is capital, and  $T_i$  is land. Land is in fixed supply, but labor and capital are adjustable. The production function is assumed to have constant returns to the three inputs, meaning that  $0 < \alpha < 1$ ,  $0 < \alpha < 1$ , and  $\alpha + \beta < 1$ . Assume that the return to capital, R, is determined by the world interest rate. By contrast, nominal wages  $W_i$ are set at the local level.

Indirect utility of the representative agent living in region i is given by:

$$V_i = \frac{W_i Z_i}{P_i^\beta} \tag{3.2}$$

where  $Z_i$  is the value of local amenities, and  $P_i$  denotes local nominal house prices. The parameter  $\beta$  is the expenditure share on housing. Local house prices are determined by:

$$P_i = \bar{P}_i L_i^{\gamma_i} \tag{3.3}$$

where  $\bar{P}_i$  is a constant, and  $\gamma_i > 0$  is the (inverse) elasticity of housing supply with respect to the number of workers in the city.

### **3.**a

Figure 1 and 2 show the distribution of wages (conditional on education, age, etc.) and house prices across cities in the United States. In light of the indirect utility function in Equation (3.2), explain why these distributions might be ineffcient. What assumptions do you need to make?

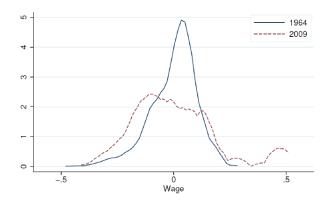


Figure 1: Conditional wages

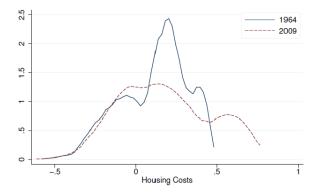


Figure 2: Housing costs

Answer: In an equilibrium with free mobility, individuals would be indifferent between living in the different cities. Assuming the existence of a representative agent, such that  $V_i = V$  in equilibrium, we get the following equilibrium condition:

$$V = \frac{W_i Z_i}{P_i^\beta} \Leftrightarrow \frac{P_i^\beta}{Z_i} V = W_i = MPL_i$$

This expression shows that differences across cities in real house prices  $\left(\frac{P_i^{\beta}}{Z_i}\right)$  may cause nominal wages to differ. The intuition is that workers need to be compensated through higher wages if house costs are high. High housing costs may therefore keep workers from moving to cities with higher wages, and thus higher marginal products of labor. The consequence is that aggregate output is smaller than it would be if real house prices were the same across cities. In this way, local policies that restrict the supply of housing may cause aggregate output to be inefficiently

low.

Under the assumptions of free mobility and the existence of a representative agent, and under the assumption that the conitiondal wages and housing costs in Figure XX and XX are empirical counterparts of  $W_i$  and  $\frac{P_i^{\beta}}{Z_i}$ , the efficient distributions would have no variance: wages and housing costs should be the same across cities. The dispersion of these variables in the figures can under these assumptions be interpreted as a measure of misallocation.

# **3.**b

Show that the (inverse) labor demand in region i is given by:

$$L_{i} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}}\frac{A_{i}}{W_{i}^{1-\eta}}\right)^{\frac{1}{1-\alpha-\eta}}T_{i}$$
$$L_{i} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}}\frac{A_{i}}{W_{i}^{1-\eta}}\right)^{\frac{1}{1-\alpha-\eta}}T_{i}$$

Answer: Profit maximization implies the following first order conditions:

$$\alpha A_i L_i^{\alpha - 1} K_i^{\eta} T_i^{1 - \alpha - \eta} = W_i$$

and

$$\eta A_i L_i^{\alpha} K_i^{\eta - 1} T_i^{1 - \alpha - \eta} = R$$

Combining these expressions yields:

$$\frac{\alpha}{\eta} \frac{K_i}{L_i} = \frac{W_i}{R}$$
$$\Leftrightarrow K_i = \frac{W_i}{R} \frac{\eta}{\alpha} L_i$$

Plug this expression for capital into the first order condition for labor:

$$W_{i} = \alpha A_{i} L_{i}^{\alpha-1} K_{i}^{\eta} T_{i}^{1-\alpha-\eta}$$

$$W_{i} = \alpha A_{i} L_{i}^{\alpha-1} \left(\frac{W_{i}}{R} \frac{\eta}{\alpha} L_{i}\right)^{\eta} T_{i}^{1-\alpha-\eta}$$

$$L_{i}^{1-\alpha-\beta} = A_{i} \left(\frac{\alpha}{W_{i}}\right)^{1-\eta} \left(\frac{1}{R} \frac{\eta}{\alpha}\right)^{\eta} T_{i}^{1-\alpha-\eta}$$

$$L_{i} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}} \frac{A_{i}}{W_{i}^{1-\eta}}\right)^{\frac{1}{1-\alpha-\eta}} T_{i}.$$

**3.c** 

Under the assumption of perfect worker mobility, show that equilibirum employment in a city is given by:

$$L_i = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}V^{1-\eta}}A_i T_i^{1-\alpha-\eta} \left(\frac{Z_i}{\bar{P}_i^{\beta}}\right)^{1-\eta}\right)^{\frac{1}{1-\alpha-\eta+\beta\gamma_i(1-\eta)}}$$

where V is common across regions.

Answer: Equilibrium implies that workers are indifferent between living in the different regions, meaning that  $V_i = V$ . Plug this condition, and Equation (3.3) into the indirect utility function:

$$W_i = V \frac{P_i^\beta}{Z_i} = V \frac{\bar{P}^\beta L_i^{\beta \gamma_i}}{Z_i}$$

This expression is the labor supply function. Use it to substitute for local wages in the labor demand function to arrive at the desired expression:

$$L_{i} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}} \frac{A_{i}}{\left(V\frac{\bar{P}^{\beta}L_{i}^{\beta\gamma_{i}}}{Z_{i}}\right)^{1-\eta}}\right)^{\frac{1}{1-\alpha-\eta}} T_{i}$$

$$L_{i}^{1+\frac{\beta\gamma_{i}(1-\eta)}{1-\alpha-\eta}} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}V^{1-\eta}}A_{i}\left(\frac{Z_{i}}{\bar{P}^{\beta}}\right)^{1-\eta}\right)^{\frac{1}{1-\alpha-\eta}} T_{i}$$

$$L_{i} = \left(\frac{\alpha^{1-\eta}\eta^{\eta}}{R^{\eta}V^{1-\eta}}A_{i}T_{i}^{1-\alpha-\eta}\left(\frac{Z_{i}}{\bar{P}^{\beta}}\right)^{1-\eta}\right)^{\frac{1}{1-\alpha-\eta+\beta\gamma_{i}(1-\eta)}}$$

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# **3.**d

Everything else being equal, what is the elasticity of city size  $(L_i)$  with respect to the local productivity level  $(A_i)$ ? Explain why the effect of an increase in  $A_i$  might be different across cities, and relate your answer to the descriptive statistics in Table 1 (reproduced from Hsieh and Moretti, 2019).

	1964	2009
log Employment		
New York, San Francisco, San Jose	2.89	2.55
Rust Belt Cities	1.63	0.96
Southern Cities	.82	1.14
Other Large Cities	2.68	2.23
log Residual Wage		
New York, San Francisco, San Jose	.041	.465
Rust Belt Cities	.072	121
Southern Cities	038	037
Other Large Cities	.010	.046
log TFP		
New York, San Francisco, San Jose	3.81	7.14
Rust Belt Cities	2.77	1.14
Southern Cities	1.14	1.95
Other Large Cities	3.36	3.68
log Housing Price		
New York, San Francisco, San Jose	.409	.610
Rust Belt Cities	.125	104
Southern Cities	128	.106
Other Large Cities	.225	.333
log Amenities		
New York, San Francisco, San Jose	.094	174
Rust Belt Cities	040	049
Southern Cities	065	026
Other Large Cities	.034	.020

## Table 1: Employment, Average Wages, TFP, Housing Prices, and Amenities

*Note:* The sample includes 220 metropolitan areas observed in both 1964 and 2009. There are 37 Rust Belt Cities, 86 Southern Cities, and 19 Other Large Cities. The table presents the employment-weighted average of each group of cities relative to the weighted average in all 220 cities in the year. Residual wage controls for educational attainment (high school drop-out, high school, college), race, gender, age,

**Answer:** The elasticity can be derived as

$$\frac{\partial \ln L_i}{\partial \ln A_i} = \frac{1}{1 - \alpha - \eta + \beta \gamma_i (1 - \eta)}$$

The elasticity is only affected by one parameter that varies across cities, namely  $\gamma_i$ , the (inverse) elasticity of housing supply with respect to the number of workers in the city. A large  $\gamma_i$  implies that even small changes in the population size of a city will substantially increase house prices because the supply of housing does not respond by much to increased demand. Demand for housing increases when  $A_i$  increases, but if  $\gamma_i$  is high, the increase in  $A_i$  will be capitalized in higher house prices rather than reflected in increases in employment.

Table 1 shows that TFP increased in New York, San Francisco, and San Jose compared to other cities over the period 1964-2009. The higher productivity level increased wages, but not employment, indicating that housing is supply constrained in the three cities. This interpretation is consistent with both the increases in house prices, and with the fact that New York and the Bay Area area widely known for their restrictive housing policies. Note also that the very light regulation of new construction in Southern cities is reflected in the table: Southern cities experienced the largest population growth over the period, but their house prices only increased modestly.